Use of Discriminant Analysis in Counseling Psychology Research

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Discriminant analysis is a technique for the multivariate study of group differences. More specifically, it provides a method of examining the extent to which multiple predictor variables are related to a categorical criterion, that is, group membership. Situations in which the technique is particularly useful include those in which the researcher wishes to assess which of a number of continuous variables best differentiates groups of individuals or in which he or she wishes to predict group membership on the basis of the discriminant function (analogous to a multiple regression equation) yielded by the analysis. The method is also useful as a follow-up to a significant analysis of variance. In this article, I describe the method of discriminant analysis, including the concept of discriminant function, discriminant score, group centroid, and discriminant weights and loadings. I discuss methods for testing the statistical significance of a function, methods of using the function in classification, and the concept of rotating functions. The use of discriminant analysis in both the two-group case and the multigroup case is illustrated. Finally, I provide a number of illustrative examples of use of the method in the counseling literature. I conclude with cautions regarding the use of the method and with the provision of resources for further study.

The technique of discriminant analysis, developed by R. A. Fisher (1936), is one method for the multivariate study of group differences. When used for explanatory purposes, discriminant analysis is particularly appropriate when one wishes (a) to describe, summarize, and understand the differences between or among groups, (b) to determine which of a set of continuous variables best captures or characterizes group differences, (c) to describe the dimensionality of group differences (much like factor analysis describes the dimensionality of a set of continuous variables), (d) to test theories that use stage concepts or taxonomies, and (e) to examine the nature of group differences following a multivariate analysis of variance (MANOVA; Borgen & Seling, 1978).

Probably the most frequent applications of discriminant analysis are for predictive purposes, that is, for situations in which it is necessary or desirable to classify subjects into groups or categories. The results of a discriminant analysis allow the prediction of group membership based on the best linear composite or combination of predictor scores. Discriminant analysis is analogous to multiple regression in that both involve prediction from a set of continuous predictor variables (sometimes designated independent variables) to a criterion. The major difference between them is that multiple regression predicts to a continuous criterion variable (sometimes designated the dependent variable), whereas discriminant analysis predicts to a categorical criterion, that is, group membership. Thus, given multiple predictor variables, multiple regression would be the appropriate method of analysis if the dependent variable were continuous, and discriminant analysis would be appropriate if the dependent variable were categorical, with two or more levels.

Some examples of research questions that would be appropriate for the application of discriminant analysis in the educational or vocational area include the following: (a) the major variables that distinguish successful and unsuccessful employees in a work setting or organization; (b) the differential characterization of students who do and those who do not successfully complete a given education program, for example, a PhD program in Counseling Psychology; and (c) the characteristics of women who pursue careers in traditionally male-dominated fields versus those who pursue careers in traditionally female-dominated areas.

In other areas of counseling research, discriminant analysis might be an appropriate method for comparing premature terminators in counseling with those who complete counseling, suicide-prone individuals who do with those who do not ultimately attempt suicide, participants in cigarette-smoking cessation programs who do with those who do not suffer relapses after completing a treatment program, and participants in marriage counseling who do with those who do not stay married after completion of counseling.

Discriminant analysis has also been suggested as a follow-up to MANOVA, which is a method used to examine group differences on a set of dependent variables. In contrast to the use of multiple t tests, the use of MANOVA provides a test of the existence of group differences across all dependent variables simultaneously. A statistically significant multivariate F indicates the likely presence of group differences, but follow-up analyses are necessary to discern the nature or sources of the differences.

Typically, researchers use separate univariate F tests as a follow-up to a significant MANOVA, but some investigators (Borgen & Seling, 1978; Bray & Maxwell, 1982; Huberty, 1975a; Tatsuoka, 1971) suggest that discriminant analysis may have some advantages over separate F tests. One advantage of discriminant analysis as a follow-up to MANOVA is that...
it makes it possible for researchers to avoid the experiment-wise error inherent in repeated univariate tests by providing for simultaneous examination of the variables. Further, it provides information concerning the dimensionality of group differences and may thus provide a more parsimonious explanation of the data. More specifically, assume that univariate tests indicated that the groups differed significantly on 6 of the 10 variables studied; discriminant analysis might yield two meaningful discriminant functions, each of which included 3 of the 6 significant dependent variables. In a case like this, the use of two functions versus the use of six variables would lead to more parsimonious description of group differences.

There are problems with the use of discriminant analysis for this purpose, particularly when the variables are more highly intercorrelated; these problems and their solution are discussed in more detail in the subsequent section on interpreting the results of a discriminant analysis. For a detailed discussion of the use of discriminant analysis as a follow-up to MANOVA, see Bray and Maxwell (1982).

Notice that each of the above uses of discriminant analysis would yield information of both theoretical and applied interest. That is, discriminant analysis can contribute to the understanding of the nature and extent of group differences and thus to an understanding of the dynamics of behavior and behavior change. In addition, it results in an equation, known as the discriminant function, by which group membership can be predicted. This information might be used in new samples to identify high-risk individuals for whom special interventions might be warranted. For example, the predictive function could be used to identify those at high risk for premature termination of counseling, to predict dropout from an educational program, or to predict relapse after a smoking-cessation program. (Again note the similarity of this application of discriminant analysis to the predictive uses of a multiple regression equation. The close relation between discriminant analysis and linear multiple regression is discussed below.)

Before the method and results of a discriminant analysis are described, it may be useful to compare the discriminant analysis with other approaches to similar research problems. Discriminant analysis is related to a whole class of methods, including regression and MANOVA, that are based on the general multivariate linear model (see Bock, 1975; Borgen & Seling, 1978). Distinctions among the methods concern the research questions they address, the number and types of variables for which they are appropriate, and their special uses.

A first major type of research question is whether or not groups differ on variables of interest. In the univariate case, group differences are examined with *t* tests or one-way analyses of variance (ANOVAS). In the multivariate case, group differences are best examined with the related methods of Hotelling's $T^2$ statistic, MANOVA, or discriminant analysis. All three of these methods are preferable to the use of multiple *t* tests or multiple ANOVAS because they control the experiment-wise error rate, that is, the overall risk of Type I error. Use of these methods allows us to answer the question of whether or not there are significant multivariate differences between two or more groups (although it should be noted that use of Hotelling's $T^2$ statistic is only appropriate in the two-group case). The need for the use of discriminant analysis as a follow-up to MANOVA was emphasized in the previous section, but MANOVA should also be viewed as one of several methods for the multivariate study of group differences.

In relation to the other methods, discriminant analysis is unique in its provision of information concerning the dimensionality of group differences, but it has the disadvantage, like multiple regression, of being a maximization procedure (to be discussed subsequently).

An understanding of discriminant analysis can probably best be conveyed by a discussion of its conceptual and mathematical similarity to multiple regression. Discriminant analysis, like multiple regression, provides the researcher with a linear equation with beta weights indicating the relative importance of each variable in predicting the criterion. In multiple regression the criterion is a continuous variable, whereas in discriminant analysis group membership is the criterion. In both cases, the weights are determined mathematically to maximize predictability of the criterion. In discriminant analysis the weights yielded are those that maximally differentiate or separate the groups. The limitations of discriminant analysis resulting from its maximizing characteristic will be discussed later, but the reader's knowledge of the limitations of multiple regression should provide useful background.

Data Analysis and Interpretation

Nature of the Data

The data used in a discriminant analysis include scores on two or more variables for two or more groups. The groups can be formed on the basis of demographic characteristics (e.g., sex, race, marital status), intellectual or personality attributes (e.g., Holland interest types, gifted vs. average intelligence groups), or actual behavior (e.g., being successful vs. being unsuccessful in school, work, or a treatment program, continuing vs. dropping out from counseling). The variables are those the researcher views as potentially important in understanding the nature of group differences; usually they are measured as continuous variables, but discrete variables may also be used on occasion. The variables are called discriminant or discriminator variables (e.g., Brown and Tinsley, 1983), but they are equivalent to predictor variables or independent variables when used for the prediction of group membership.

A variety of computer programs are available for the data analysis, including the Statistical Package for the Social Sciences (SPSS) program DISCRIMINANT (Nie, Hull, Jenkins, Steinbrenner, & Bent, 1975), Statistical Analysis System SAS DISCRIM (SAS, Inc., 1985), and Biomedical Data Package (BMDP)'s program for stepwise discriminant analysis (Dixon, 1985). Although the programs yield similar types of information, there are minor variations in the types of statistics provided. As in the case when multiple regression is used, the researcher must decide on the strategy by which variables are to be entered into the predictive equation; options usually include forward selection and stepwise selection.


Discriminant Function

Discussion of the mathematical computations of a discriminant analysis is beyond the scope of this article, but can be found in Morrison (1976) or Tatsuoka (1971). A discriminant analysis is designed to enable the researcher to search for the linear equation that will maximize differences between the groups. Recall the general form of a linear equation, $Y = bX + a$, and the form in linear multiple regression:

$$
\hat{Y} = b_1X_1 + b_2X_2 + \ldots + b_pX_p + a,
$$

where the $bs$ are the weights applied to the variables $X$, the $a$ is a constant (reflecting the $Y$-intercept of the regression line) and the $Y$ is the continuous variable to be predicted. On the basis of the familiar principle of least squares, the weights are selected so as to minimize the sum of squared errors, that is, the squared errors in the prediction of $Y$ from $Y$. Similarly, the linear equation that is the basis of discriminant analysis is called a discriminant function and takes the following analogous form:

$$
D = b_1X_1 + b_2X_2 + \ldots + b_pX_p + a,
$$

where $D$ is the categorical variable to be predicted, specifically group membership. The objective of the use of the discriminant analysis is to form a linear equation for each group that maximizes the differences between the weighted group means, the $Ds$, also called group centroids (see subsequent discussion). Another way of describing this, for those more comfortable with the ANOVA paradigm, is that the weights are chosen to maximize the ratio of the between-groups sum of squares to the within-groups sum of squares. In effect, variables on which the groups differ are generally weighted more heavily, and those variables on which the groups are similar receive smaller weights. Note that the technique emphasizes group differences and deemphasizes group similarities.

A discriminant analysis thus results in a discriminant function, or set of beta weights to be applied to the variables, in which the weights indicate the importance of each variable in contributing to group differences. The method also provides information regarding the statistical significance of the function as a whole and of the individual variable weights (statistical significance is described in the section on interpreting a discriminant function). A statistically significant function, and its associated beta weights, are used for explanatory purposes, that is, to enhance understanding of the nature of group differences. For predictive purposes, discriminant scores are calculated for each individual and compared with group centroids to determine probabilities of group membership. These concepts are discussed in the next section.

Just as a multiple regression equation can be used to calculate a predicted score on the criterion for each subject, a discriminant function can be used to calculate an individual's discriminant score. Again, on the basis of the general formula for a linear equation ($Y = bX + a$), the individual's scores on each variable are multiplied by the corresponding discriminant weight. Multiplication of raw scores by unstandardized weights would result in a discriminant score in the same units as the original variables. Multiplication of standard scores by standardized weights would yield discriminant scores in standard score units. Application of Equation 2 to the calculation of the standardized discriminant score for the $i^{th}$ individual would result in the following:

$$
D_i = b_{1i}X_{1i} + b_{2i}X_{2i} + b_{3i}X_{3i} + \ldots + b_{pi}X_{pi}.
$$

(Classification)

A discriminant analysis enables the investigator to make a prediction of group membership for each individual in the sample. Classification is based on the concepts of the discriminant score and the group centroid, as discussed previously; very simply, classification of an individual case involves calculation of the individual's discriminant score and comparison of it with the centroid of each group studied. The centroid to which the individual's score is closest is the group to which he or she is predicted to belong.

The reader may note that predicted group membership can be compared with actual group membership in the sample in which the function was calculated. The percentage of correct predictions based on the function can be compared with the percentage that can be predicted correctly with other strategies; if no other alternatives are available, the percentage correctly classified can be compared with the percentage of correct predictions expected on the basis of chance.

If the groups are equal in size, the percentage of correct predictions based on chance is equal to $1/k$, where $k$ is the number of groups. For example, if we have three equal-sized groups, the chances of correctly classifying any given individual are .333. When sample sizes are unequal, there are two ways of estimating the percentage that could be correctly classified by chance. The first, which assumes that all correct predictions are equal in value, is to use the formula $n/N$,
where \( n \) is the size of the largest group and \( N \) is the total sample size. For example, assume that we have 300 successes and 100 failures in a job training program. If we make a prediction of success for every individual, we will be correct 75% of the time, that is, \( 300/400 = .75 \) by using the above formula. However, predicting success for all 400 cases doesn't help with the problem at hand, which is to predict in advance those individuals who will fail. An alternative formula that assumes a comparable rate of error across groups is

\[
P_1a_1 + p_2a_2 + \ldots + p_Ka_K.
\]

(5)

In the formula, the \( p \) values refer to the proportion of cases in the sample belonging to each group, the values of \( a \) refer to the proportion actually classified as belonging to that group, and \( k \) is the number of groups. Assume that in the earlier example, the discriminant function led to the prediction of 60% successes and 40% failures. By inserting these values into the formula we would have a chance rate of correct prediction of (.75) (.60) + (.25) (.40) = .55. Note that the latter value is considerably less than the value of .75 that was based on the prediction of success for all cases.

The actual percentage of correct predictions can be compared statistically to that expected on the basis of chance by using the \( z \) test for the difference between proportions (Glass & Stanley, 1970). Thus, the ability of a discriminant function to make a statistically significant improvement in the accuracy of classification can be assessed. It is essential to note that cross-validation is absolutely necessary if the investigator wishes to apply the function to the prediction of group membership in subsequent samples of individuals (versus those in the sample in which the function was originally developed). As has already been mentioned, discriminant analysis is a maximization procedure, which means that it capitalizes on sample-specific error. In order to assess the probability of correct classification in any new group, the discriminant weights must be applied in a new sample and the actual percentage of correct predictions determined. This new percentage is a better approximation of the long-term predictive accuracy of the function. (Methods of cross-validation are discussed below.)

This discussion provides a somewhat simplified but conceptually meaningful explanation of how a discriminant score is assigned to a group in discriminant analysis. For complete accuracy, it should be noted that the actual statistical procedure derives a probability of group membership and takes into account other information, including information regarding base rates (also called prior or unconditional probabilities of group membership) and conditional probabilities, which are used in the formula for Bayes's theorem (e.g., see Hays, 1981) to yield a posterior probability, that is, the probability of membership in a given group for an individual with score \( X \). A case is classified, on the basis of its discriminant score, in the group for which the posterior probability is largest; in other words, a case is assigned to the most likely group on the basis of its discriminant score.

In cases in which the prior probabilities of correct classification (base rates) diverge greatly from 50%, it may be more difficult to improve upon the accuracy of classification possible through classification of every individual into the largest group. Assume, for example, a population in which past history shows that 90% of the people are successful; if a guess of success for each new member is made, the guess will be correct 90% of the time. It will be difficult to achieve greater accuracy in the prediction of success than that which is possible by using base rates alone, although the function may still considerably increase our accuracy in predicting failures.

The example that follows provides an example of this situation. See Brown and Tinsley (1983), Cronbach and Gleser (1965), Meehl and Rosen (1955), Taylor and Weiss (1972), and Wiggins (1973) for more extensive discussions of these and other issues involved in classification.

If the discriminant function is to be used for predictive purposes in new populations, it is essential that the sample specificity of the discriminant analysis, and thus its tendency to overestimate the accuracy of classification, be considered. There are several methods of cross validation, including the following (Dillon & Goldstein, 1984; Brown & Tinsley, 1983): (a) cross-validation using a holdout sample; (b) double cross-validation; and (c) what has been called the jackknife, U-method, or "leaving-one-out" method.

In the holdout method of cross-validation, the sample is split in two. One part, usually at least half the group of subjects, is used to derive the initial discriminant function, and the weights are then applied to the classification of the subjects in the second or holdout sample. Although this represents an unbiased method of estimating the true misclassification rate, it requires large sample sizes if reasonably sound initial discriminant functions are to be derived. In double cross-validation, the total sample is divided in half. Separate discriminant analyses are performed on each sample, and the results are cross-validated on the other sample.

In the third method, one observation at a time is held out, the discriminant function is estimated on the basis of the remaining observations, and that discriminant function is used to classify the held-out observation. This process is repeated until all observations have been classified. Error rates can be determined on the basis of the cumulative findings. The jackknife method is available on BMDP, and discussions of its use are offered by Efron (1983) and Dillon and Goldstein (1984).

**Examples**

In order to illustrate the ideas presented up to this point, as well as to introduce the idea of statistical significance of a discriminant function, results from a study of predictors of students' continuation in college mathematics studies are presented. This study illustrates the use of discriminant analysis with a dichotomous criterion variable, that is, a criterion consisting of two groups. An example presented later illustrates use of the method with a polytomous criterion, that is, one with three or more groups.

Mathematics has been called the "critical filter" of career development (Sells, 1982; Sherman, 1982, p. 428) because lack of high school and college mathematics serves to filter people out from many potentially interesting career possibilities. Thus, it is important to understand the factors that
influence individuals' plans to continue math studies in both high school and college because of the important role that knowledge of mathematics plays in making a range of educational and career options available to them. As part of a larger study of the correlates of math anxiety in college students (Bander & Betz, 1981; N. E. Betz, 1978), a discriminant analysis of predictors of intent to continue in college mathematics was performed.

The dependent or predictor variables used in the analysis were variables, including sex, math anxiety, math ability, prior math background, and interest in choosing a major in one of the sciences, postulated to be related to the extent to which college students continued to study mathematics. The independent or grouping variable was intent to continue math studies. The subjects were students enrolled in freshman math courses. The results of this analysis are described below.

Interpretation: Significance testing. Table 1 shows the discriminant weights or coefficients which, when multiplied by the individual’s scores on the variables or by the group means, will yield the discriminant score or group centroid, respectively. However, in order to interpret the results of a discriminant analysis, the investigator’s first concern should be that of the statistical significance of the function yielded.

There are several methods of testing the significance of a discriminant function. One common method, based on the familiar concepts of between-groups, within-groups, and total sums of squares, tests the null hypothesis that the weighted group means (centroids) are equal by using Wilks’s lambda statistic. Wilks’s lambda is the ratio of within-groups variance to total variance (sum of squares) and is therefore the percentage of variance in discriminant scores not explained by group membership. It is useful to transform the variance ratio into lambda, because lambda can be transformed into a statistic that has a chi-square distribution. As shown in the note at the bottom of the table, the value of Wilks’s lambda for the function calculated was .91, distributed as a χ²(5, N = 376) = 33.6, p < .001. This is interpreted as indicating that the null hypothesis of equality of group means can be rejected at the .001 level.

An eigenvalue can also be calculated; the eigenvalue is the ratio of between-groups to within-groups sum of squares, so that large eigenvalues indicate good functions. The canonical correlation R_c is a measure of the degree of association between the discriminant scores and group membership, and is equivalent to the eta derivable from ANOVA (Klecka, 1975). In the two-group case, the canonical correlation is equal to a point-biserial correlation between the continuously distributed discriminant scores and dichotomous group membership (Klecka; Thorndike, 1978).

As is the case with other statistical methods, statistical significance may not always lead to practical significance, particularly when sample sizes are large. The present case provides a good example of this because the actual values of Wilks’s lambda, although statistically significant (the N was 376), are unimpressive. Further, the eigenvalue of .09 and the R_c of .29 (see the note to Table 1) indicate that the actual percentage of variance accounted for by the function is unimpressive even though the group centroids differ significantly. In such cases, the usefulness of the function for practical purposes may rest on its ability to classify individuals into groups (see discussion later in this section.)

If the overall function is statistically significant, then the weights, the contributions of the individual variables to the differentiation of the groups, can be evaluated for significance. Methods of testing the significance of the discriminant weights include a univariate F calculated for each variable (equal to the value of F for a one-way ANOVA with the same number of groups) and Wilks’s lambda for the univariate case. When variables are considered individually, lambda is the ratio of within-groups to total sum of squares. A lambda of 1 occurs when all group means are equal, but values closer to 0 indicate that most of the total variability can be attributed to between-groups differences. Thus, smaller values of Wilks’s lambda indicate variables that better differentiate the groups.

From an examination of the weights shown in Table 1, it is evident that smaller values of Wilks’s lambda correspond to the same variables for which the F is statistically significant. Specifically, the weights corresponding to sex, math anxiety, and interest in a major in science were statistically significant, indicating that these variables make significant contributions to the prediction of intent to continue studies in math. Substantive interpretation of the direction of weights can be performed by using the tables of group means and standard deviations (which should always be provided in studies reporting the findings of a discriminant analysis) and by using information about the direction of coding of categorical variables and of scoring of continuous variables. (The reader should note that it is always necessary to provide numerical values for categorical variables, e.g., by coding continuers 2 and noncontinuers 1. The analysis can be done only if all variables have numerical values, even though these values are not meaningful in a numerical sense. This is often referred to in

Table 1
Results of Discriminant Analysis of Variables Related to Intent to Continue Mathematics Coursework Among College Freshmen (N = 376)

<table>
<thead>
<tr>
<th>Predictor variable</th>
<th>Standardized discriminant function coefficient</th>
<th>Wilks’s lambda</th>
<th>F(1, 374)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Sex</td>
<td>-.75</td>
<td>.953</td>
<td>18.6**</td>
</tr>
<tr>
<td>Relative freedom</td>
<td>.58</td>
<td>.980</td>
<td>7.5*</td>
</tr>
<tr>
<td>from math anxiety</td>
<td>-.51</td>
<td>.999</td>
<td>0.13</td>
</tr>
<tr>
<td>ACT math score</td>
<td>-.04</td>
<td>.997</td>
<td>1.17</td>
</tr>
<tr>
<td>Amount of high school</td>
<td>.35</td>
<td>.986</td>
<td>5.40*</td>
</tr>
<tr>
<td>Interest in a science major</td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

Note: In interpreting the direction of the weights, it may be noted that intent to take more math was coded 2, whereas plans to discontinue math were coded 1. For sex, male was coded 1, female 2. Higher scores on the math anxiety scale were indicative of less anxiety (more positive attitudes) and are, thus, shown as “freedom from math anxiety” to facilitate interpretation. For the function as a whole, Wilks’s lambda = .91, distributed as a χ² statistic with 5 degrees of freedom and equal to 33.6, p < .001, the eigenvalue = .09, and R = .29. Group centroids were .14 and -.60 for continuers and noncontinuers, respectively. ACT = American College Test.

*p < .01   **p < .001
computer manuals and elsewhere as dummy coding.) In the present case, positive coefficients are associated with students’ intention to take more math courses, and negative coefficients with their intention to discontinue their study of math. The negative value for sex indicates the association of being a male (coded 1) versus being a female (coded 2) with the intention to continue studies in math. Thus, the conditions of being a male, being less math-anxious, and planning to major in science significantly differentiated students who did from those who did not plan to continue studies in math.

Although statistically significant weights indicate variables that contribute significantly to group differences, intercorrelations among variables reduce the extent to which the weights and their statistical significance can be considered unambiguous. Discriminant coefficients are equivalent to partial regression coefficients. Thus, if the predictors are intercorrelated, one predictor may have received most of the weight, whereas another may have received little weight (see Bock, 1975, pp. 417-420 for examples; see also Bray & Maxwell, 1982, p. 345). Further, only the standardized weights can be compared in absolute size, because the magnitude of the unstandardized weights varies with different units of measurement of the variables.

In addition to its usefulness in interpreting the variables contributing to the understanding of group differences, discriminant analysis can also be used for classification, as illustrated in this example. As was mentioned earlier, the discriminant weights, when multiplied by an individual’s standard scores on each variable, yield a discriminant score, and when multiplied by the score means for a group, yield the group centroid. The function used to calculate the standardized discriminant score for the rth individual would be as follows:

\[ D_i = -.75 \text{ (Sex)}, + .58 \text{ (Math Anxiety)}, - .51 \text{ (American College Test [ACT] Math Score)}, - .04 \text{ (Amount of High School Math)}, + .35 \text{ (Science Major Plans)} \]

Similarly, the group centroid for the continuers would involve multiplying the weights by the group means on the variables.

In the present case, the group centroids were .14 for the continuers and -.60 for the noncontinuers.

Figure 1 shows the distribution of individual scores around their own group centroid. Assume, for example, that two subjects, A and B, are selected at random from the sample, without the experimenter’s knowledge of the group to which each belongs. The discriminant score for Subject A is calculated as .30 and that for Subject B as -.90. The discriminant score for Subject A is closer to the centroid of the continuers and that for Subject B is closer to that of the centroid of the noncontinuers. Accordingly, we would predict that Subject A would continue and Subject B would not continue studies in math. For further information about the geometric representation of individual discriminant scores around their group centroids and the relation of that to classification, see Tatsumaki (1971).

Results concerning the accuracy of the discriminant function in classifying continuers and noncontinuers in math studies are presented by the cross-tabulation shown in Table 2. As shown in the table, the function resulted in correct predictions being made for 66% of the subjects; 64% of the continuers and 71% of the noncontinuers were correctly classified. As was mentioned previously, this percentage may be compared to the percentage of correct predictions that would be possible if alternative strategies were used. If it is assumed that our only alternative strategy is chance prediction, this study shows that the two methods of determining the percentage of correct predictions on the basis of chance will yield different conclusions regarding the usefulness of the function in classification.

Because of the disparate size of the two groups, the largest percentage of correct classifications based on chance would be obtained by assigning every individual to the continuer group. In accordance with the formula \( n/N \), where \( n \) is the size of the largest group and \( N \) is the total sample size, the result was 303/376, or 81%. Clearly our obtained value of 66% is inferior to the result obtained by the latter method of calculating chance accuracy. However, consider the fact that although we correctly predicted 81% of the total number of cases, by predicting that all subjects would be in the continuer group we misclassified all 73 noncontinuers. Thus, for the
Table 2

**Hit Rates Using a Discriminant Function to Predict Intent to Continue in College Mathematics**

<table>
<thead>
<tr>
<th>Actual group</th>
<th>Predicted group</th>
<th>Continue</th>
<th>Dropout</th>
<th>Total</th>
</tr>
</thead>
<tbody>
<tr>
<td>Plans to continue</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>No.</td>
<td>195</td>
<td>108</td>
<td>303</td>
<td></td>
</tr>
<tr>
<td>%</td>
<td>64%</td>
<td>36%</td>
<td>81%</td>
<td></td>
</tr>
<tr>
<td>Does not plan to continue</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>No.</td>
<td>21</td>
<td>52</td>
<td>73</td>
<td></td>
</tr>
<tr>
<td>%</td>
<td>29%</td>
<td>71%</td>
<td>19%</td>
<td></td>
</tr>
<tr>
<td>Total</td>
<td>216</td>
<td>160</td>
<td>376</td>
<td></td>
</tr>
<tr>
<td>No.</td>
<td>57%</td>
<td>43%</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

Note. Values on the diagonal are “hits” and are in italic type. There are a total of 247 hits, or 66%. Conversely, the 129 misses account for 34% of the cases.

latter group, our success rate based on chance was 0 versus the 71% success rate that it is possible to obtain by using the function. Because one of our research objectives was to identify the probable noncontinuers in order to give them special help to encourage them to continue studies in math, it is important to predict the noncontinuers as accurately as the continuers. Thus, the second formula, \( p; a_1 + p; a_2 + \ldots + p; a_k \), could be used. In the present case, we would have this value: (81%) (57%) + (19%) (43%) = 54% expected on the basis of chance. Not only was our obtained value of 66% an improvement over this value of 54%, but in using the \( z \)-test for the difference between two proportions (e.g., Glass & Stanley, 1970, p. 313), we found that it is a statistically significant improvement. Use of the function would lead to a large number of misclassifications, especially among students planning to continue math studies who were predicted to drop out, but 71% of the potential dropouts would be correctly identified. Thus, the function would be useful in identifying students in need of support to continue math studies.

**Additional Issues in Interpreting Discriminant Functions**

In addition to providing information about discriminant weights, a discriminant analysis usually includes a canonical structure matrix, that is, a matrix of the correlations of each variable with each function; these correlations are known as canonical variate correlations or as discriminant loadings (Bray & Maxwell, 1982). These loadings are conceptually similar to factor loadings (Huberty, 1975a) and can therefore be used to interpret the dimensionality of group differences (Borgen & Sehing, 1978). Some statisticians contend that these loadings are more stable on cross-validation than arc discriminant weights and thus may be safer to interpret. However, studies comparing the stability of weights versus loadings on cross-validation (Barcikowski & Stevens, 1975; Huberty, 1975b; Thorndike & Weiss, 1973) do not consistently suggest the superiority of one over the other.

In terms of practical uses, discriminant weights, which reflect the unique contribution of a variable to a composite, are useful in determining whether or not to retain a variable in the set of discriminators to be used. Discriminant loadings, because they reflect the shared variance between a variable and an underlying composite, may be especially useful for interpreting the substantive nature of the composite or function.

Also useful in the interpretation of a discriminant analysis, particularly one using three or more groups and resulting in two or more significant functions, is the fact that the functions can be rotated to improve clarity and simplicity of structure. A set of discriminant functions is analogous to the principal axes obtained at the beginning of a factor analysis. Thus, like principal axes, the functions may be more interpretable and meaningful after rotation.

In essence, the initial discriminant loadings are rotated to form new factors, which are simply linear combinations of the old factors. In rotation for simple structure, the rotated function is to correlate highly with a few predictors and at a level near zero with the rest. Specifically, the rotated (as opposed to the initial) values of the standardized discriminant coefficients and loadings will typically be closer to 0 or 1, thereby improving the interpretability of the function and group differences. However, the total discriminatory power of the model and the relative position of the groups remains the same after rotation. For further discussion of the rationale for and methods of rotation, particularly as they relate to uses in counseling psychology, see Lunneborg and Lunneborg (1978). For an example of the use of rotation, see Anderson, Wahlberg, and Welch (1969).

The next example illustrates the use of rotated factor loadings in the interpretation of the results of a discriminant analysis. The example, from a study by Martin and Bartol (1986), was designed to investigate differences among students enrolled in six different areas of concentration in a master of business administration program. In one part of the study, discriminant analysis was used to examine the extent to which scores on the six Holland (1973, 1985) themes differentiated students in different areas of concentration. Thus, there were six predictors (the six Holland themes, as measured by Holland's, 1978, Vocational Preference Inventory) and six groups to be differentiated. The maximum number of discriminant functions that can be yielded in a discriminant analysis is one fewer than the number of groups \( k \) or the number of discriminant variables, whichever is smaller. Thus, in the two-group case, only one discriminant function is yielded. In the present example using six groups, five functions are yielded. Each successive function is formed so that it is uncorrelated with previous functions and so that it maximizes the ratio of residual between-groups to within-groups variability. In other words, the first function extracted accounts for the maximum possible between-groups variance. Later functions attempt to account for leftover or residual between-groups variance. Although \( k - 1 \) functions may be yielded, the number of statistically significant and therefore interpretable functions may range from zero to \( k - 1 \).

Table 3 shows the rotated discriminant structure matrix (the loadings) and the group centroids resulting from Martin
Among Groups of MBA Students

Discriminant Analysis of Holland Theme Differences

Table 3
Group Centroids and Discriminant Structure Matrix for Discriminant Analysis of Holland Theme Differences Among Groups of MBA Students

<table>
<thead>
<tr>
<th>Group or variable</th>
<th>Discriminant function</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Group centroids</td>
</tr>
<tr>
<td></td>
<td>1</td>
</tr>
<tr>
<td>Accounting</td>
<td>0.381</td>
</tr>
<tr>
<td>Finance</td>
<td>0.327</td>
</tr>
<tr>
<td>Information systems</td>
<td>0.081</td>
</tr>
<tr>
<td>Management</td>
<td>-0.616</td>
</tr>
<tr>
<td>Management science</td>
<td>-0.985</td>
</tr>
<tr>
<td>Marketing</td>
<td>0.195</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Discriminant structure matrix</th>
</tr>
</thead>
<tbody>
<tr>
<td>Realistic</td>
</tr>
<tr>
<td>Investigative</td>
</tr>
<tr>
<td>Social</td>
</tr>
<tr>
<td>Conventional</td>
</tr>
<tr>
<td>Enterprising</td>
</tr>
<tr>
<td>Artistic</td>
</tr>
</tbody>
</table>

Note: Data from a study by Martin & Bartol (1986). MBA = master of business administration.
* The variables are scores on the Holland (1973, 1985) themes as measured by the Vocational Preference Inventory (Holland, 1978).

The range of possible applications of discriminant analysis within counseling psychology is large and diverse, and in the following section some examples of recently published or completed research that used this method are provided. Note that these examples are only a small sample of the wealth of possibilities.

First, in the area of research on educational and career development, E. L. Betz (1982) used discriminant analysis to examine differences between three groups of women on the five needs postulated by Abraham Maslow, that is, security-safety, social, autonomy, esteem, and self-actualization. The three groups of women were homemakers, women employed in professional-managerial occupations, and women employed in clerical and sales occupations. The analysis yielded one statistically significant function that differentiated homemakers from working women and was characterized by higher scores on the higher order needs among the working women and higher scores on the lower needs among the homemakers.

Beutell and Brenner (1986) used discriminant analysis to study gender differences in work values. Their discriminant analysis of the 25 work values measured by Manhardt’s (1972) job orientation scale, with gender as the grouping variable, yielded a significant discriminant function. Of the 25 values (called job outcomes by Manhardt), 18 contributed significantly to the differentiation of the sexes. Variables particularly characteristic of the female group included higher scores on the values of working with congenial associates, using one’s education in a job, of having a feeling of accomplishment, of being respected by others, and of being able to work independently. Values for which higher male scores contributed significantly to the function included higher income, job security, and the opportunity for advancement. Tinsley and Kass (1980) investigated the degree to which different psychological needs were satisfied by different leisure activities.

Utz (1983) compared three groups of students with vocational problems on three measures; the three groups were (a) a group of students who sought counseling at the counseling center; (b) a group of students enrolled in a course on career planning; and (c) a group of students who were undecided about careers but who had not sought help. Discriminant analysis indicated several differences among the groups, including more positive attitudes toward counselors and counseling among the students who had sought help at the center.

In an important study of minority group concerns, LaFromboise (1986) used discriminant analysis to investigate the degree to which low expectations of self-efficacy were related to the extreme underrepresentation of American Indian women in U.S. colleges and universities (LaFromboise, 1984). LaFromboise constructed four 10-item efficacy scales that were used to assess expectations of personal efficacy with respect to academic success, career advancement, ability to manage stress, and ability to survive socially in a white-dominated collegiate environment. Discriminant analysis using the four subscale scores indicated the behavioral domains most important in differentiating American Indian from white women, and analyses of the items within each scale...
indicated the areas of perceived deficit versus competence characterizing each group.

Note the utility of LaFromboise’s study in both understanding the obstacles faced by American Indian women and in the design of interventions targeted toward those areas of behavioral performance in which the efficacy expectations of American Indians are particularly low relative to those of Anglo women. LaFromboise also notes that discriminant analysis is especially useful in the study of cross-cultural issues because of the diagnostic information it provides when subjects are misclassified. Exploration of the background and experience of American Indians whose discriminant scores were more similar to the Anglo centroid could contribute to attempts to assist American Indian women.

In the therapeutic area, Sloat, Leonard, and Gutsch (1983) compared drug users and nonusers on the scales of the 16 Personality Factor Questionnaire. The discriminant analysis indicated that 8 of the 16 scales significantly differentiated users from nonusers. Not only do these findings contribute to the understanding of the personality correlates of drug abuse, but the classification methods of discriminant analysis could be used to compare the discriminant scores of individuals in high risk populations with user and nonuser centroid scores to derive a predicted group membership, thus identifying individuals at risk for abuse.

Discriminant analysis can be very useful in some types of theory testing and explication. For example, the E. L. Betz (1982) study described earlier used the method to examine the applicability of Maslow’s need theory to women’s career development. Other stage theories, for example those of Super, Perry, Chickering, Erikson, or Kohlberg, could be further explicated by examination of the degree to which various individual difference variables differentiated individuals at various stages postulated by the theory. Discriminant analysis is one useful method for the study of individual and group differences and, as was mentioned earlier, is useful as a follow-up examination of the univariate effects contributing to a significant multivariate $F$. Finally, the research studies used as examples at the beginning of this article all represent areas for the fruitful application of discriminant analytic techniques.

Discussion and Summary

Cautions in the Use of Discriminant Analysis

In order to use any statistical method, it is necessary to understand the mathematical and distributional assumptions inherent in the technique and to ensure that the characteristics of the data do not violate these assumptions. In the case of discriminant analysis, it is assumed (a) that there is linearity in the relation between predictors, (b) that the continuous variables come from a multivariate normal population, and (c) that the covariance matrices for the groups are equal. Therefore, the researcher should first check the distributions of the individual variables for significant departures from normality and the bivariate scatterplots for deviations from linearity. (Normality of the individual distributions is a necessary though not sufficient requirement for multivariate normality.) The equality of group covariance matrices can be tested by using Box’s $M$ test (cf. Norusis, 1985), which tests the null hypothesis of equality of the matrices.

Recently several new approaches to checking the assumptions of multivariate normality and homoscedasticity have been developed, for example, Hawkins’s (1981) procedure, which is available on the BMDP and SAS software packages. The effects of violation of these assumptions, which include reductions in the accuracy of prediction and decreased stability in discriminant weights, are reviewed in detail by Dillon and Goldstein (1984). Dillon and Goldstein also discuss methods of handling cases in which some of the predictors are discrete rather than continuous.

Discriminant analysis, like multiple regression, is a maximization procedure; that is, it locates the set of weights (the linear equation) that maximizes the correlation between the predictor set and group membership. All maximization procedures capitalize on sample-specific covariation, and discriminant analysis is no exception. Note that to ensure that a discriminant function is valid and generalizable beyond the sample in which it was initially derived, it should be cross-validated to determine the stability of the weights and the actual predictive accuracy of the equation. Thorndike (1978) provides an example of how the size of discriminant loadings and of the canonical correlation can shrink dramatically after cross-validation.

In order to minimize the capitalization on sample-specific error, it is useful to perform an $a$ priori test for profile separation by using Hotelling’s $T^2$ (see Harris, 1975; Morrison, 1976). Although a statistically significant value of $T^2$ cannot be considered unambiguous, because this statistic is susceptible to sample-specific error, performance of further discriminant analyses after obtaining a nonsignificant $T^2$ test result is likely to extract sample-specific rather than generalizable group differences.

A final caution arises from the conceptual basis of discriminant analysis, specifically its emphasis on difference and its deemphasis on similarity. Thus, the method exacerbates a sometimes unfortunate tendency to emphasize difference rather than similarity in the field of psychology as a whole. For example, findings of gender or racial similarities are almost never viewed as conceptually interesting, yet if they are viewed within the larger context of a society which fosters gender and racial differences, they gain a unique and major import. Thus, the theoretical importance and meaning of human difference versus similarity should be a consideration of our research.

Resources for Further Study

For readers who would like additional information about discriminant analysis, a variety of references is available. For a readable and informative overview of multivariate methods, including discriminant analysis, see Weiss’s chapter in Dunnette’s (1976) Handbook of Industrial and Organizational Psychology. Brown and Tinsley (1983) also provide a readable
and understandable summary of discriminant analysis, with an emphasis on its use in leisure research. Most major texts on multivariate or correlational methods have chapters on discriminant analysis; some of the more helpful include Tatsuoka's (1971) *Multivariate Analysis*, Thorndike's (1978) *Correlational Procedures for Research*, Marascuilo and Levin's (1983) *Multivariate Statistics in the Social Sciences*, and Dillon and Goldstein's (1984) *Multivariate Analysis*. Most computer packages are a good source of reviews of the methods available; some of them are quite extensive. See, for example, Norusis's (1985) *SPSS Advanced Statistics Guide* for a detailed discussion of discriminant analysis and its interpretation. Goldstein and Dillon (1978) review six computer programs that are helpful in the performance of discriminant analyses.

At a more advanced level are Morrison's (1976, 1983) texts on multivariate methods. Huberty's (1975a) review of discriminant analysis, Tatsuoka's (1971) excellent discussion of the geometric representation of the results of discriminant analysis, particularly when there are two or more significant functions, Lunneborg and Lunneborg's (1978) discussion of rotation, Borgen and Seling's (1978) comparison of discriminant analysis to univariate ANOVAs following MANOVA, and Harris's (1975) treatment of Hotelling's $T^2$ statistic, including discussion of its use for profile analysis.

**Summary**

Discriminant analysis provides information that contributes to an increased understanding of the nature, extent, and dimensionality of group differences, as well as to the prediction of group membership for purposes of selection, placement, and intervention, and for testing stage and taxonomic theories. The utility of this method, like many others, has not been fully appreciated in counseling psychology. It is hoped that this introduction will increase readers' interest in and ability to appropriately use discriminant analysis.

**References**


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